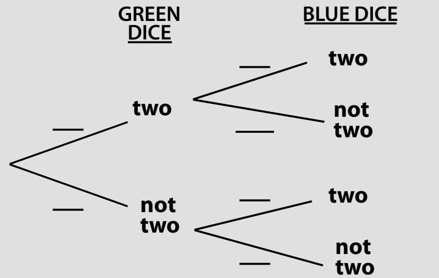
D6 PROBABILITY 2 & COMBINED MEANS

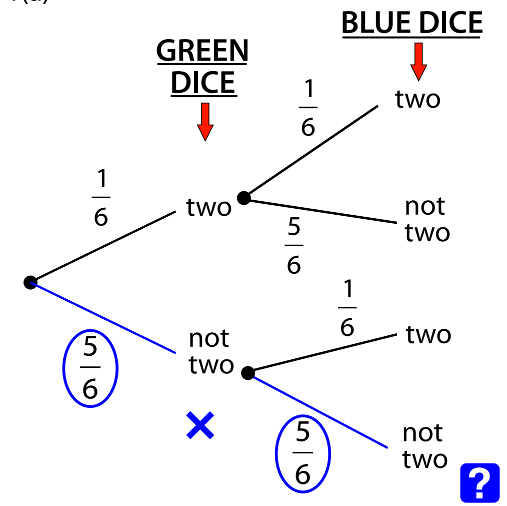
■ IF P (EVENT A) = $\frac{1}{4}$ P(**NOT** EVENT A) = 1 − P (EVENT A) = 1 − $\frac{1}{4}$ = $\frac{3}{4}$

■ P (EVENT A **AND THEN** EVENT B) = **P (A) × P (B)**

(1) A fair (unbiased) green dice and a fair blue dice are each thrown once.
(a) Complete the tree diagram.
Calculate the probability:
(b) that a 2 is not rolled at all
(c) that exactly one 2 only is rolled
(B) (4) ★★



1(a)



Probability (of a 2 on a fair dice) = $\frac{1}{6}$

P(**not** getting 2) = 1 - $\frac{1}{6}$ = $\frac{5}{6}$

ⓘ This is the same diagram if ONE dice is thrown 2 times. The headings would be changed to 1st THROW and 2nd THROW.

To work out probabilities at the END of 2 touching branches: multiply the probabilities together

(b) P (a 2 is not rolled at all)	=	P(NOT 2	**AND THEN**	NOT 2)		
	=	P(NOT 2)	×	P(NOT 2)		
	=	$\frac{5}{6}$	×	$\frac{5}{6}$	=	$\frac{25}{36}$ ✓

(c) P (one 2 only is rolled) = P(2 **AND THEN** NOT 2) **OR** P(NOT 2 **AND THEN** 2)

$= (\frac{1}{6} \times \frac{5}{6}) + (\frac{5}{6} \times \frac{1}{6}) = \frac{10}{36} = \frac{5}{18}$ ✓

COMBINED MEANS

COMBINED MEANS
THINK: GROUP 'TOTALS'

IF GROUP A = GROUP B + GROUP C

[eg class of pupils = boys + girls]

mean of group A = $\frac{\textbf{total of A values}}{\textbf{no. of A values}}$ = $\frac{\textbf{(total of B values + total of C values)}}{\textbf{(no. of B values + no. of C values)}}$

TO FIND COMBINED MEANS, YOU HAVE TO FIND GROUP TOTALS FIRST

group total = group mean × no. in group

COMBINED MEANS: TYPE 1 — FIND THE MEAN OF THE WHOLE GROUP

(1) 20 CDs are sold, with a mean average price of £12.80.
5 more CDs are sold – these have a mean price of £13.
What is the mean average price of all the 25 CDs ? (B) (3) ★

20 CDs:	Total Cost	=	20	×	£12.80	=	£256	
5 CDs:	Total Cost	=	5	×	£13.00	=	£ 65	
All 25 CDs	Total Cost	=	£256	+	£65	=	£ 321	
Mean of all 25 CDs		=	$\frac{\text{total cost of 25 CDs}}{25}$		=	$\frac{321}{25}$	=	£12.84 ✓

COMBINED MEANS: TYPE 2 — FIND THE MEAN OF A SMALL GROUP

(2) 30 Runners in a 6 mile race have a mean finishing time of 52 minutes.
The top 25 runners have a mean time time of 48 minutes.
Find the mean time of the last group of 5 runners. (B) (3) ★

All 30 RUNNERS:	total of all 30 times	=	30	×	52	=	1560 mins
Top 25 runners:	total of top 25 times	=	25	×	48	=	1200 mins
Last 5 runners:	total of last 5 runners	=	total of all times − total of 1st 25				
		=	1560	−	1200	=	360
	mean time of last 5 runners	=	360	÷	5	=	72 mins ✓

N15 PROPORTIONALITY

- STEP 1 - IDENTIFY : PROPORTIONAL OR INVERSELY PROPORTIONAL
- STEP 2 - REPLACE PROPORTIONAL SIGN " $\propto$ " WITH " = k "
- STEP 3 - FIND k : USE THE 2 VALUES GIVEN IN THE QUESTION
- STEP 4 - WRITE DOWN THE EXPRESSION
- STEP 5 - USE THE EXPRESSION TO CALCULATE OTHER VALUES

IDENTIFY QUESTION TYPE

QUESTION		STEP 1	STEP 2
F is (directly) proportional	to x	$F \propto x$	$F = k\,x$
F is inversely proportional or: F varies inversely	to x	$F \propto \frac{1}{x}$	$F = k\,\frac{1}{x}$
F is (directly) proportional	to x squared	$F \propto x^2$	$F = k\,x^2$
F is inversely proportional or: F varies inversely	to x squared	$F \propto \frac{1}{x^2}$	$F = k\,\frac{1}{x^2}$

(1) V is proportional to the cube of r. V = 6 when r = 2.
(a) Express V in terms of r.
(b) Work out the value of V when r = 6
(c) Work out the value of r when V = 48

A 6 ☆☆

(a) " V is **PROPORTIONAL** to the cube of r "

STEP 1 $V \propto r^3$

STEP 2 $V = k\,r^3$

STEP 3 Find k : Use V = 6 and r = 2 (given in question)

$V = k\,r^3$

$6 = k\,2^3$

$6 = k \times 8$

$k = \frac{6}{8} = \frac{3}{4}$

STEP 4 so expression is $V = \frac{3}{4}\,r^3$ ✓

(b) Find V when r = 6 $V = \frac{3}{4}\,r^3$

$V = \frac{3}{4} \times 6^3 = \frac{3}{4} \times 216 = 162$ ✓

(c) Find r when V = 48: $V = \frac{3}{4}\,r^3$

$48 = \frac{3}{4} \times r^3$

$48 \times \frac{4}{3} = r^3$

$\frac{192}{3} = r^3$

$64 = r^3$

$r = \sqrt[3]{64} = 4$ ✓

(2) F varies inversely as the square of d. F = 8 when d = 5.
(a) Express F in terms of d.
(b) Work out the value of F when d = 4
(c) Work out the positive value of d when F = 72

A 6 ☆☆

(a) F is **INVERSELY PROPORTIONAL** to the square of d

STEP 1 $F \propto \frac{1}{d^2}$

STEP 2 $F = k\,\frac{1}{d^2}$

STEP 3 Find k : Use F = 8 and d = 5 (given in question)

$8 = k\,\frac{1}{5^2} = k\,\frac{1}{25}$

$k = 25 \times 8 = 200$

STEP 4 so expression is $F = k\,\frac{1}{d^2} = 200\,\frac{1}{d^2}$ so $F = \frac{200}{d^2}$ ✓

(b) Find F when d = 4 $F = \frac{200}{d^2}$

$F = \frac{200}{4^2} = \frac{200}{16} = \frac{100}{8} = \frac{50}{4} = 12.5$ ✓

(c) Find d when F = 72 $F = \frac{200}{d^2}$

$72 = \frac{200}{d^2}$ so $72 \times d^2 = \frac{200}{d^2} \times d^2$

$72 \times d^2 = 200$

$\frac{72 \times d^2}{72} = \frac{200}{72}$

$d^2 = \frac{200}{72} = \frac{100}{36} = \frac{25}{9}$

$d = \sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3} = 1\,\frac{2}{3}$ ✓

A20* A STAR: FACTORISING

FACTORISING USING: 'Completing the Square Method'

- **Factorising quadratics into:** $(x + m)^2 + n$

$$1x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$$ ← LEARN !

- **Find Maximum or Minimum Value:** when $(\ )^2$ term $= 0$

eg $(x - 4)^2 - 9$ Minimum at $(4, -9)$

$x = +4$ so bracket $= 0$; y coordinate $= -9$

(1) $x^2 + 8x = (x + 4)^2 - n$ for all values of x.
Find the value of n. A* ② ★★

Find n: $1x^2 + 8x + 0$ compare with: $1x^2 + bx + c$

$a = 1$, $b = 8$, $c = 0$

$$1x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$$

$$x^2 + 8x = \left(x + \frac{8}{2}\right)^2 - \frac{8^2}{4} + 0 \qquad \frac{8^2}{4} = \frac{64}{4} = 16$$

so $(x+4)^2 - \mathbf{n} = (x+4)^2 - 16$ so $\mathbf{n} = 16$ ✓

(2) $y = f(x)$, where $f(x) = x^2 - 6x + 16$.
This can be written in the form $(x - p)^2 + r$ for all values of x.
(a) Find the values of p and r.
(b) Write down the coordinates of the minimum point M. A* ④ ★★

(a) $1x^2 - 6x + 16$ compare with: $1x^2 + bx + c$

$a = 1$, $b = -6$, $c = 16$

$$1x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$$

negative signs !

$$= \left(x + \frac{(-6)}{2}\right)^2 - \frac{(-6)^2}{4} + 16$$

$$= (x - 3)^2 - \frac{36}{4} + 16$$

$$= (x - 3)^2 - 9 + 16$$

$(x - \mathbf{p})^2 + \mathbf{r} = (x - 3)^2 + 7$ so $\mathbf{p} = 3$ & $\mathbf{r} = 7$ ✓

(b) Minimum point occurs when $(x - 3)^2 = 0$. Minimum at $x = 3$, $y = 7$ ✓

(1) $x^2 - 4nx = (x - 2n)^2 - k$ for all values of n and x.
(a) Express k in terms of n.
(b) Find the minimum value of $x^2 - 4nx$, give your answer in terms of n.
(c) State the value of x for which this minimum value occurs, in terms of n. A* ⑤ ★

(a) LHS: $1x^2 - 4nx$ compare with: $1x^2 + bx + c$

$a = 1$, $b = -4n$, $c = 0$

$$1x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$$

$$x^2 - 4nx = \left(x + \frac{-4n}{2}\right)^2 - \frac{(-4n)^2}{4} + 0$$

$$x^2 - 4nx = (x - 2n)^2 - \frac{16n^2}{4}$$

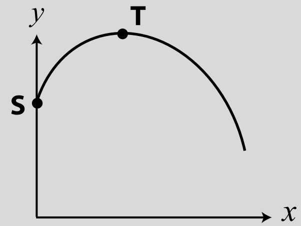
so $(x - 2n)^2 - \mathbf{k} = (x - 2n)^2 - 4n^2$ so $\mathbf{k} = 4n^2$ ✓

(b) The minimum value of $x^2 - 4nx$ is found using: $(x - 2n)^2 - 4n^2$

$(x - 2n)^2$: Minimum occurs when this term $= 0$. $-4n^2$: Minimum y Value. Minimum Value $= -4n^2$ ✓

(C) The minimum value: x value is when $(x - 2n) = 0$ so $x = 2n$ ✓

(2)



$$y = 20 - \frac{(x - 6)^2}{3} \qquad 0 \le x \le 13$$

S is the point where the graph meets the y axis. T is the maximum value. Find the coordinates of:
(a) point S (b) point T A* ③ ★

(a) Point S occurs on y axis when $x = 0$

$$y = 20 - \frac{(x - 6)^2}{3}$$ substitute $x = 0$

$$y = 20 - \frac{(0 - 6)^2}{3} = 20 - \frac{(-6)^2}{3}$$

$$y = 20 - \frac{36}{3}$$

$y = 20 - 12 = 8$ So S is the point (0, 8) ✓

(b) $20 - \frac{(x - 6)^2}{3}$ So Maximum Value $= 20$

20: Maximum y Value. $\frac{(x-6)^2}{3}$: Maximum occurs when this term $= 0$, at $x = 6$

Coordinates of maximum value: (6, 20) ✓

SHAPE ☆☆ **GCSE Maths Revision Cards™** HIGHER 2008

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GRAPHS TRANSFORMATIONS TYPES

$f(x + a)$	$f(x - a)$
Horizontal Shift (Translation): whole graph moves $-a$ (left), y SAME; vector: $\begin{bmatrix} -a \\ 0 \end{bmatrix}$	**Horizontal Shift (Translation):** whole graph moves $+a$ (right), y SAME; vector: $\begin{bmatrix} a \\ 0 \end{bmatrix}$

$f(x) + a$	$f(x) - a$
Vertical Shift (Translation): whole graph moves $+a$ (up), x SAME; vector: $\begin{bmatrix} 0 \\ a \end{bmatrix}$	**Vertical Shift (Translation):** whole graph moves $-a$ (down), x SAME; vector: $\begin{bmatrix} 0 \\ -a \end{bmatrix}$

$a\,f(x)$

Vertical stretch $\times a$ (vertical), SAME (x)

(One way enlargement in the y axis direction)

$f(ax)$

Horizontal stretch SAME (y), $\times \frac{1}{a}$ (horizontal)

(One way enlargement in the x axis direction)

$-f(x)$

Horizontal Reflection (x axis)

Reflection in x axis *Negative (y coordinate)*

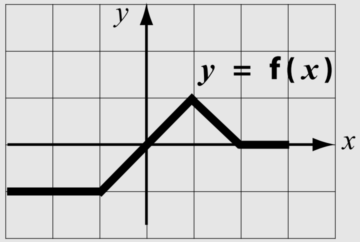
$f(-x)$

Vertical Reflection (y axis)

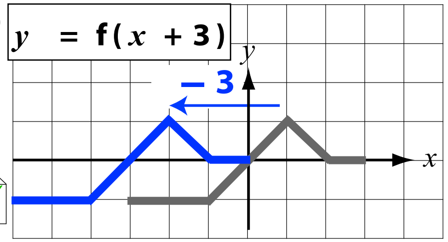
Reflection in y axis *Negative (x coordinate)*

(1) $y = f(x)$ is shown opposite. A* 4 ☆
Sketch the following graphs:

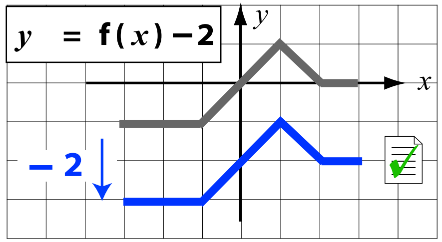
(a) $y = f(x + 3)$ (b) $y = f(x) - 2$
(c) $y = f(-x)$ (d) $y = -f(x)$
(e) $y = f(2x)$ (f) $y = 2f(x)$



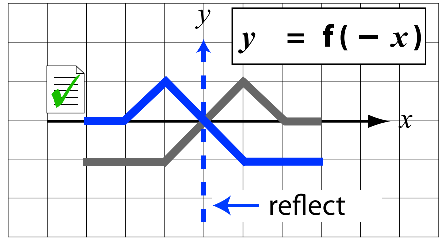
(a) $y = f(x + 3)$



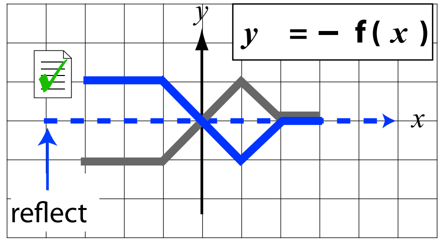
(b) $y = f(x) - 2$



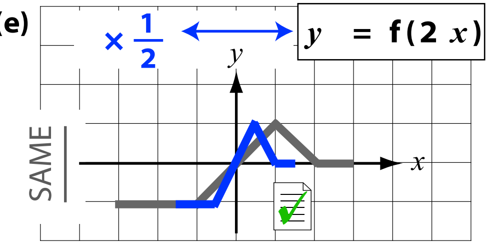
(c) $y = f(-x)$



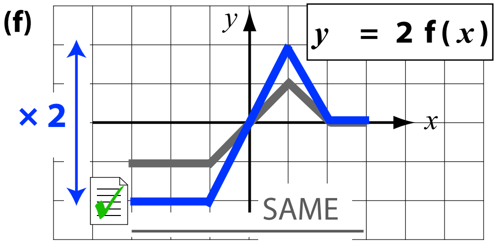
(d) $y = -f(x)$



(e) $y = f(2x)$

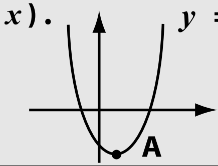


(f) $y = 2f(x)$



(2) A is the point $(2, -5)$ on the curve $y = f(x)$. A* 4 ☆
Write down the coordinates of A when:

(a) $y = f(x - 2)$ (b) $y = f(x) + 4$
(c) $y = f(-x)$ (d) $y = f(2x)$



(a) $y = f(x - 2)$ → $+2$
x coordinate $+2$ so: $(4, -5)$

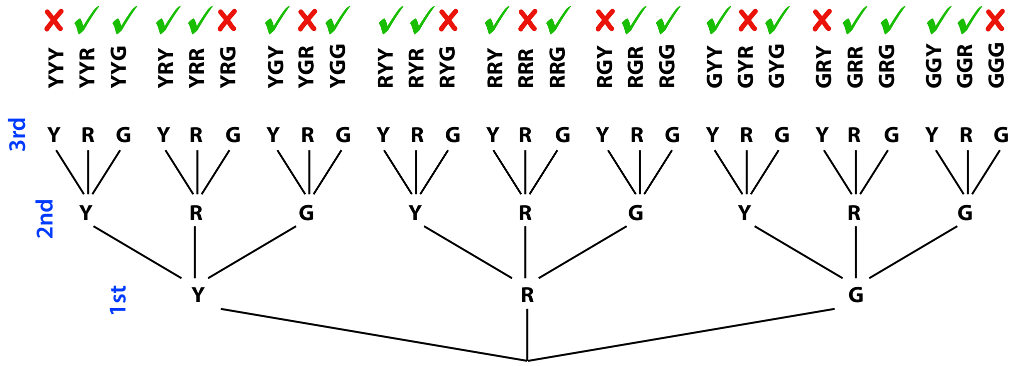
(b) $y = f(x) + 4$ ↑ $+4$
y coordinate $+4$ so: $(2, -1)$

(c) $y = f(-x)$ Reflect in y axis
Negative (x coordinate): $(-2, -5)$

(d) $y = f(2x)$ $\times \frac{1}{2}$ ↔
$\frac{1}{2}$ the x coordinate: $(1, -5)$

A* PROBABILITY: 3 TRIALS

(1) 2 yellow, 3 red and 5 green counters are placed in a bag. A counter is selected at random, its colour noted and then replaced into the bag. 3 counters are selected using this method. Calculate the probability that 2 of the counters are exactly the same colour. A* 5 ☆



⚠ To have exactly 2 of the same colour, with 3 trials, means 1 counter **must** be a different colour

3 different ways for each combination of: 2 colours the same, and 1 different

P(2Y and 1R)	=	P(YYR)	+	P(YRY)	+	P(RYY)
	=	$(\frac{2}{10}\times\frac{2}{10}\times\frac{3}{10})$	+	$(\frac{2}{10}\times\frac{3}{10}\times\frac{2}{10})$	+	$(\frac{3}{10}\times\frac{2}{10}\times\frac{2}{10})$
	=	**3** $(\frac{2}{10}\times\frac{2}{10}\times\frac{3}{10})$	=	$\frac{36}{1000}$		

P(2Y and 1G)	=	3 × P(Y) × P(Y) × P(G)	=	$3\ (\frac{2}{10}\times\frac{2}{10}\times\frac{5}{10})$	=	$\frac{60}{1000}$
P(2R and 1Y)	=	3 × P(R) × P(R) × P(Y)	=	$3\ (\frac{3}{10}\times\frac{3}{10}\times\frac{2}{10})$	=	$\frac{54}{1000}$
P(2R and 1G)	=	3 × P(R) × P(R) × P(G)	=	$3\ (\frac{3}{10}\times\frac{3}{10}\times\frac{5}{10})$	=	$\frac{135}{1000}$
P(2G and 1Y)	=	3 × P(G) × P(G) × P(Y)	=	$3\ (\frac{5}{10}\times\frac{5}{10}\times\frac{2}{10})$	=	$\frac{150}{1000}$
P(2G and 1R)	=	3 × P(G) × P(G) × P(R)	=	$3\ (\frac{5}{10}\times\frac{5}{10}\times\frac{3}{10})$	=	$\frac{225}{1000}$

P (exactly 2 of the same colour) = total probabilities of all combinations above

$$= \frac{36}{1000} + \frac{60}{1000} + \frac{54}{1000} + \frac{135}{1000} + \frac{150}{1000} + \frac{225}{1000}$$

$$= \frac{660}{1000} = \frac{66}{100} = \frac{33}{50}$$ ✓

A* PROBABILITY : INDEPENDENT EVENTS

(2) The probability that Geoff is late for a party is 0.3. The probability that Hazel is late for a party is 0.4. The probability Geoff and Hazel are both late for the same party is 0.7. Show whether or not these are 2 independent events. A* 2 ☆

Events G and H are independent if:	P(G)	×	P(H)	=	P(G and H)
	0.3	×	0.4	≠	0.7
				=	0.12
So the events are NOT independent ✓					

A* PROBABILITY : PROOF

(3) A bag contains blue, red and black counters. There are n blue counters. There are 6 more red counters than blue counters. The number of black counters is equal to the number of red and blue counters.
(a) Show that the number of counters in the bag is equal to 4 (n + 3).
(b) Tony takes 1 counter from the bag, notes its colour and then returns it to the bag. Sam also takes 1 counter from the bag in the same way. The probability that Tony's counter is blue and Sam's counter is not blue is $\frac{2}{13}$. Prove that $7n^2 - 36n - 288 = 0$. A* 6 ☆

<u>Blue Counters:</u>	<u>Red Counters:</u>	<u>Black Counters:</u>
n (given)	= 6 more than blue = **n + 6**	= blue + red = n + n + 6 = **2n + 6**

(a) total no. of counters = blue + red + black = n + (n + 6) + (2n + 6)
= 4n + 12 = 4 (n + 3) ✓

(b) P (Blue) $= \frac{\text{no. of blue counters}}{\text{total no. of counters}} = \frac{n}{4(n+3)}$

P (Not Blue) $= \frac{\text{no. of red and black counters}}{\text{total no. of counters}} = \frac{n+6 \ + \ 2n+6}{4(n+3)} = \frac{3n+12}{4(n+3)}$

P (Blue and then Not Blue) = P (Blue) × P (Not Blue)

$$= \frac{n}{4(n+3)} \times \frac{3n+12}{4(n+3)} = \frac{n(3n+12)}{4^2(n+3)^2}$$

$$\frac{n(3n+12)}{4^2(n+3)^2} = \frac{3n^2+12n}{16(n^2+6n+9)} = \frac{3n^2+12n}{16n^2+96n+144}$$

We are told P (Blue and then Not Blue) $= \frac{2}{13}$, So: $\frac{3n^2+12n}{16n^2+96n+144} = \frac{2}{13}$

× 13 and × $(16n^2 + 96n + 144)$ to both sides gives:

$$13 \times (3n^2 + 12n) = 2 \times (16n^2 + 96n + 144)$$
$$39n^2 + 156n = 32n^2 + 192n + 288$$
$$39n^2 + 156n - (32n^2 + 192n + 288) = 0$$
$$7n^2 - 36n - 288 = 0$$ ✓